
MS
3
تمارين في
التكامل
Integration
المحدد وغير المحدد

احسب التكاملات التالية

1) $\int \sqrt[3]{x}\, dx$

2) $\int (x^2 + 2x)^2 dx$

3) $\int x\sqrt{x+1}\, dx$

4) $\int \frac{\sin x}{\cos x} dx$

5) $\int x\,|x-1|\, dx$

6) $\int \sqrt{2+2\cos 2x}\, dx$

7) $\int 2x .\cos^2 x .\sin x\, dx$

8) $\int x\sqrt{x-1}\, dx$

9) $\int \sqrt{x-1}\, dx$

10) $\int \sin^3 x\, dx$

11) $\int \frac{x^2}{(x^3+8)^3} dx$

12) $\int \left(\frac{x-1}{x} \right) e^{\frac{1}{x}} dx$

13) $\int \sin^3 x \cos x \, dx$

14) $\int \frac{x}{\sqrt{3x+1}} dx$

15) $\int \sqrt{1-x^3} \, dx$

16) $\int \sqrt[3]{x^2} \, dx$

17) $\int \frac{1}{\sqrt{x^3+1}} dx$

18) $\int \cot x \, dx$

19) $\int \frac{1}{2x+\sqrt{x}} dx$

20) $\int \tan x \, dx$

21) $\int \frac{-2}{x^2-4x+3} dx$

$$\int \frac{e^{x}-1}{e^{x}+1}\,dx$$ (22

$$\int \sqrt{x^{2}-4x+3}\,dx$$ (23

$$\int \frac{\sqrt{x}}{x-1}\,dx$$ (24

$$\int \left|\frac{x+3}{x+1}\right|\,dx$$ (25

$$\int e^{\frac{x-1}{x}}\,dx$$ (26

$$\int e^{\frac{1}{x}}\,dx$$ (27

$$\int \frac{\ln x}{x}\,dx$$ (28

$$\int \frac{1}{x\ln x}\,dx$$ (29

$$\int \ln x\,dx$$ (30

$$\int \frac{-1}{x^{2}+x}\,dx$$ (31

$$\int \frac{\ln^3(x)}{x}\,dx \quad (32$$

$$\int \sqrt{\sin x}\,dx \quad (33$$

$$\int \frac{x}{x^n+1}\,dx \quad (34$$

$$\int \frac{\ln x}{x^3}\,dx \quad (35$$

$$\int x^n \ln x\,dx \quad (36$$

$$\int \sin x \cos\left(x-\frac{\pi}{4}\right)dx \quad (37$$

$$\int \frac{e^{\frac{1}{x}}}{x^2}\,dx \quad (38$$

$$\int \frac{1}{\sin 2x}\,dx \quad (39$$

$$\int \left|x^2-4x\right|dx \quad (40$$

$$\int \frac{2x-1}{x-1}\,dx \quad (41$$

$$\int x e^{x}\,dx \quad (42$$

$$\int \left(\frac{x-1}{x}\right) e^{\frac{1}{x}}\,dx \quad (43$$

$$\int \frac{1}{x^{2}} e^{\frac{1}{x^{2}}}\,dx \quad (44$$

$$\int \sqrt{\tan^{2} x+1}.\frac{1}{\cos x}\,dx \quad (45$$

$$\int \sqrt{1+\sin 2x}\,dx \quad (46$$

$$\int \cos^{2}\left(\sqrt{x}\right)dx \quad (47$$

$$\int \frac{\arcsin \sqrt{x}}{\sqrt{x}}\,dx \quad (48$$

$$\int x^{7} e^{x^{2}}\,dx \quad (49$$

$$\int \sqrt[5]{x}\,\ln(x)\,dx \quad (50$$

$$\int x^{3}\,chx\,dx \quad (51$$

$$\int x^{n} \sin x\,dx \quad (52$$

$$\int e^{ax}\cos(bx)\,dx \quad (53$$

$$\int \frac{(1-x)^2}{x\sqrt[3]{x}}\,dx \quad (54$$

$$\int e^{x}(\cot e^{x})\,dx \quad (55$$

$$\int \frac{\sqrt{\tan x -1}}{\cos^2 x}\,dx \quad (56$$

$$\int \sqrt{\frac{1+\ln x}{x}}\,dx \quad (57$$

$$\int \frac{\sin 2x}{\sqrt{1+\sin^2 x}}\,dx \quad (58$$

$$\int \frac{2^x.3^x}{9^x+4^x}\,dx \quad (59$$

$$\int \sqrt{\frac{e^{4x}}{\sqrt{e^x+1}}}\,dx \quad (60$$

$$\int \ln(1-x)\,dx \quad (61$$

$$\int \frac{\cos x}{\sin^2 x+4\sin x+7}\,dx \quad (62$$

$$\int \frac{1}{(\sin x+1)^2}\, d(\sin x) \quad (63$$

$$\int \frac{1}{x\sqrt{\ln x}}\, dx \quad (64$$

$$\int \frac{e^x+e^{2x}}{1-e^x}\, dx \quad (65$$

$$\int \frac{x^2}{x^6-5}\, dx \quad (66$$

$$\int \frac{2x+4}{x^4-3x^3+2x^2}\, dx \quad (67$$

$$\int \frac{1}{x^7-x}\, dx \quad (68$$

$$\int \frac{\sqrt[3]{x^2}+\sqrt[4]{x}}{\sqrt{x}+\sqrt[3]{x}}\, dx \quad (69$$

$$\int e^{3x}\cos 4x\, dx \quad (70$$

$$\int (x^2+a^2)^n\, dx \quad (71$$

$$\int \frac{1}{(2x-1)\sqrt{16x^2-12x+3}}\,dx$$ (72

$$\int \left(x^2+2x\right)dx$$ (73

$$\int \frac{1}{5+\sqrt{x+3}}\,dx$$ (74

$$\int \frac{\sin 2x}{1-\cos x}\,dx$$ (75

$$\int x\,(3x+4)^{100}\,dx$$ (76

$$\int \sin^n x\,dx$$ (77

$$\int \frac{1}{\sin^n x}\,dx$$ (78

$$\int \tan^n x\,dx$$ (79

$$\int \sqrt[3]{\frac{x+1}{x-1}}\cdot\frac{1}{x+1}\,dx$$ (80

$$\int \frac{2+\sqrt{x+1}}{(x+1)^2-\sqrt{x+1}}\,dx$$ (81

$$\int \left(x^2+4\right)^3 dx \quad (82$$

$$\int \frac{1}{x\left(x^2+1\right)\sqrt{x^2+1}}\, dx \quad (83$$

$$\int \frac{1}{x\sqrt{\left(1+x^2\right)^3}}\, dx \quad (84$$

$$\int (x+1)\sqrt{x^2+2x+2}\, dx \quad (85$$

$$\int \sin^5 x \,.\cos^2 x \, dx \quad (86$$

$$\int \frac{x^2}{(x+1)^2(x+4)^2}\, dx \quad (87$$

$$\int \frac{x^3-6}{x^3+6x^2+8}\, dx \quad (88$$

$$\int sh(3x)\, dx \quad (89$$

$$\int \frac{4}{x^4+1}\, dx \quad (90$$

$$\int \sqrt[6]{2+\sqrt{x+2}}\, dx \quad (91$$

$$\int \frac{1}{\sqrt{9-x^{2}}}dx$$ (92

$$\int \frac{x^{4}}{\left(x^{2}-1\right)\left(x+2\right)}dx$$ (93

$$\int \frac{1}{x^{2}+9}dx$$ (94

$$\int \frac{1}{\sqrt{9+x^{2}}}dx$$ (95

$$\int \frac{1}{\sqrt{x^{2}-9}}dx$$ (96

$$\int \frac{x+\sqrt[3]{x^{2}}+\sqrt[6]{x}}{x\left(1+\sqrt[3]{x}\right)}dx$$ (97

$$\int \frac{1}{(x+1)\sqrt{1+x-x^{2}}}dx$$ (98

$$\int \frac{1}{\sqrt{1+2x-3x^{2}}}\,dx \quad (99$$

$$\int \frac{\sin x}{1+\sin x}\,dx \quad (100$$

$$\int \frac{x^{3}-2x^{2}+4x-8}{x^{4}-16}\,dx \quad (101$$

$$\int \frac{\cos x}{1-\cos x}\,dx \quad (102$$

$$\int \frac{\sqrt[6]{x}+1}{\sqrt[6]{x^{7}}+\sqrt[4]{x^{5}}}\,dx \quad (103$$

$$\int \sqrt{x}\,\cos\sqrt{x}\,dx \quad (104$$

$$\int \frac{x^{6}+x^{3}-x^{2}}{x^{4}-1}\,dx \quad (105$$

$$\int \frac{x+3}{x^{3}+3x}\,dx \quad (106$$

$$\int \frac{x^2-1}{\left(x^2+1\right)\sqrt{1+x^4}}dx$$ (107

$$\int \frac{\sin^2 x}{1+\cos^2 x}dx$$ (108

$$\int \frac{1-\sqrt{x^2+x+1}}{x\sqrt{x^2+x+1}}dx$$ (109

$$\int \frac{1}{4-5\sin x}dx$$ (110

$$\int \frac{x+1}{x\left(1+xe^x\right)}dx$$ (111

$$\int \frac{\sqrt{x^3}-\sqrt[3]{x}}{6\sqrt[4]{x}}dx$$ (112

$$\int \frac{3x-7}{x^3+x^2+4x+4}dx$$ (113

$$\int \frac{2x^2-3x-3}{(x-1)\left(x^2-2x+5\right)}dx$$ (114

$$\int \frac{3x+2}{x(x+1)^3}\,dx \quad (115$$

$$\int \frac{x^5+x^4-8}{x^3-4x}\,dx \quad (116$$

$$\int \left(x^3+\frac{1}{\sqrt[3]{x}}\right)^2 dx \quad (117$$

$$\int \frac{1}{\cos^2(5x)}\,dx \quad (118$$

$$\int \frac{x}{\sqrt{2^{x^2}+3}}\,dx \quad (119$$

$$\int \frac{\arcsin x}{\sqrt{1-x^2}}\,dx \quad (120$$

$$\int x e^{2x}\,dx \quad (121$$

$$\int \frac{\sqrt{9-x^2}}{x^2}\,dx \quad (122$$

$$\int \ln\left(x+\sqrt{1+x^2}\right) dx \quad (123$$

$$\int \frac{1}{x^3+1}\,dx \quad (124$$

$$\int \frac{2+\sqrt[3]{x}}{\sqrt[6]{x}+\sqrt[3]{x}+\sqrt{x}+1}\,dx \quad (125$$

$$\int \frac{x^3}{\sqrt{x}}\,dx \quad (126$$

$$\int \frac{1}{x^2+2x+5}\,dx \quad (128$$

$$\int \sqrt{x\sqrt{x\sqrt{x\sqrt{x}}}}\,dx \quad (128$$

$$\int \frac{1}{\sqrt[5]{x}}\,dx \quad (129$$

$$\int \left(\frac{4}{\sqrt{x}}-\frac{x\sqrt{x}}{5}\right)dx \quad (130$$

$$\int \cos x\,.e^{\sin x}\,dx \quad (131$$

$$\int e^{x^2+4x+2}(x+2)\,dx \quad (132$$

$$\int \frac{1}{\left(1+x^{2}\right)\arctan x} dx \quad (133$$

$$\int \frac{\sqrt{1+\ln x}}{x} dx \quad (134$$

$$\int \ln\left(x^{2}+1\right) dx \quad (135$$

$$\int x \arctan\sqrt{x^{2}-1}\, dx \quad (136$$

$$\int \frac{1}{ch^{2}x} dx \quad (137$$

$$\int \frac{1}{sh^{2}x} dx \quad (138$$

$$\int thx\, dx \quad (139$$

$$\int \frac{1}{shx} dx \quad (140$$

$$\int \frac{1}{chx} dx \quad (141$$

$$\int th^{2}x\, dx \quad (142$$

$$\int \cos^{5} x\, dx \quad (143$$

$$\int \left(x^{3}+1\right)\cos\left(\frac{-x}{3}+7\right)dx$$ (144

$$\int \left(3x^{2}-2x-1\right)\arcsin x\,dx$$ (145

$$\int shx\,.\ln\left(chx\right)dx$$ (146

$$\int \frac{1}{2\sin^{2}x+3\cos^{2}x}dx$$ (147

$$\int x\cos^{2}x\,dx$$ (148

$$\int x^{2}\cos x\,dx$$ (149

$$\int \sqrt{x^{2}+2x}\,dx$$ (150

$$\int 2^{x}.3^{2x}.5^{3x}\,dx$$ (151

$$\int \frac{1}{\cos\left(\ln x\right)}dx$$ (152

$$\int e^{\sqrt{x}}\,dx$$ (153

$$\int \frac{x}{\sqrt{1+x^{2}}}\arctan x\,dx$$ (154

$$\int \sin^5 x \sqrt[3]{\cos^2 x}\, dx \quad (155$$

$$\int \frac{\sin x}{2^{-x+1}}\, dx \quad (156$$

$$\int \frac{\sin 2x}{\sqrt{1+\sin^2 x}}\, dx \quad (157$$

$$\int \frac{\arctan x}{1+x^2}\, dx \quad (158$$

$$\int \frac{\cos(\ln x)}{x}\, dx \quad (159$$

$$\int \frac{\tan^3 x}{\cos x}\, dx \quad (160$$

$$\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}\, dx \quad (161$$

$$\int \frac{1}{x\sqrt{1+\ln^2(x)}}\, dx \quad (162$$

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx \quad (163$$

$$\int \sqrt{1+2\cos x}\,.\sin x\,dx \quad (164$$

$$\int \frac{1}{\left(4+x^{2}\right)\left(4+x^{2}\right)}\,dx \quad (165$$

$$\int \frac{x\,\sin x}{\cos^{2} x}\,dx \quad (166$$

$$\int \frac{1}{\sqrt{1-x}\,\arcsin x}\,dx \quad (167$$

$$\int \frac{1}{\sqrt{x}\,\sqrt{1+\sqrt{x}}}\,dx \quad (168$$

$$\int \frac{1}{\sqrt{2-3x-4x^{2}}}\,dx \quad (169$$

$$\int x\,.\frac{\arctan x}{\left(1+x^{2}\right)^{2}}\,dx \quad (170$$

$$\int\left(\frac{2}{x^{2}}+\frac{6}{x\sqrt{x}}+3\right)dx$$ (171

$$\int\frac{\cos x}{\sin^{3}x-\cos^{3}x}\,dx$$ (172

$$\int\frac{1}{\sqrt[3]{\sin^{11}x\cos x}}\,dx$$ (173

$$\int\frac{1}{\sqrt[4]{5x-3}+1}\,dx$$ (174

$$\int x\sqrt{\frac{x-1}{x+1}}\,dx$$ (175

$$\int\frac{2x+\sqrt{4x-1}}{3\sqrt[4]{4x-1}+\sqrt[4]{(4x-1)^{3}}}\,dx$$ (176

$$\int x\arctan x\,dx$$ (177

$$\int e^{\arcsin x}\,dx$$ (178

$$\int\sqrt{4-x^{2}}\,dx$$ (179

$$\int x\cos^{2}x\,dx$$ (180

$$\int x\arctan\sqrt{x^{2}-1}\,dx \quad (181$$

$$\int \frac{\sqrt[3]{x}+1}{\sqrt[6]{x^{7}}+\sqrt[4]{x^{5}}}\,dx \quad (182$$

$$\int \frac{1}{3\cos x-4\sin x+2}\,dx \quad (183$$

$$\int \frac{x^{2}}{\left(x^{2}+2\right)\left(x^{2}+1\right)}\,dx \quad (184$$

$$\int \frac{x^{2}+x+x+\sqrt{x}}{\sqrt[3]{x}+\sqrt{x}}\,dx \quad (185$$

$$\int x^{2}\sqrt{4-x^{2}}\,dx \quad (186$$

$$\int \ln\left(x+\sqrt{x^{2}+1}\right)dx \quad (187$$

$$\int \frac{1}{x^{4}+1}\,dx \quad (188$$

$$\int \frac{4}{x^{4}-16}\,dx \quad (189$$

$$\int \sqrt[3]{\frac{1}{x^{2}}}\,dx \quad (190$$

$$\int \frac{x^{5}}{\left(x^{2}+4\right)^{2}}\,dx \quad (191$$

$$\int \frac{1}{x^{2}\left(1+x^{2}\right)^{2}}\,dx \quad (192$$

$$\int \frac{\tan^{2} x}{\sin^{2} x+3\cos^{2} x}\,dx \quad (193$$

$$\int sh^{5} x\,dx \quad (194$$

$$\int \frac{1}{sh^{8} x}\,dx \quad (195$$

$$\int \frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}\,dx \quad (196$$

$$\int \frac{2^{x}}{5^{\frac{x}{2}}}\,dx \quad (197$$

$$\int x^{2} e^{2x+1} dx$$ (198

$$\int \frac{\sqrt{x} + \sqrt[3]{x}}{\sqrt[4]{x^{5}} + \sqrt[6]{x^{7}}} dx$$ (199

$$\int (x^{2}+1) e^{x^{3}+3x} dx$$ (200

$$\int \frac{6}{(x-7)^{4}} dx$$ (201

$$\int \frac{2x-5}{(x^{2}-5x+7)} dx$$ (202

$$\int \frac{30x+20}{(3x^{2}+4x+2)^{6}} dx$$ (203

$$\int \frac{x-1}{(x^{2}+2x+3)^{2}} dx$$ (204

$$\int \frac{5x+3}{(x^{2}+x+3)^{3}} dx$$ (205

$$\int e^{\arcsin x} dx$$ (206

$$\int \cos(\ln x)\,dx$$ (207

$$\int (\sqrt{x}+1)e^{\sqrt{x}}\,dx$$ (208

$$\int \frac{2x^3+5x^2+4x^2-8x+4}{x+1}\,dx$$ (209

$$\int \frac{x^4-2x^3+4x^2-8x+4}{x^2+4}\,dx$$ (210

$$\int \left(\frac{1}{(1-x)^2}-\frac{1}{1-x}\right)dx$$ (211

$$\int \frac{1}{\sqrt{x}+(\sqrt{x}-2)^2}\,dx$$ (212

$$\int e^{\sin 2x}(1-2\sin^2 x)\,dx$$ (213

$$\int \sqrt{1+e^x}\,dx$$ (214

$$\int \ln|x|\,dx$$ (215

$$\int (2sh5x-3ch5x)\,dx$$ (216

$$\int x\sin(1-x^2)\,dx$$ (217

$$\int \frac{\left(a^{x}-b^{x}\right)^{2}}{a^{x}b^{x}}dx \quad (218$$

$$\int 3^{x}e^{x}dx \quad (219$$

$$\int \frac{x^{3}}{x^{8}+5}dx \quad (220$$

$$\int \frac{-\sqrt{x}+\sqrt[3]{x}}{\sqrt[6]{x^{7}}-\sqrt[4]{x^{5}}}dx \quad (221$$

$$\int \sqrt[3]{\frac{(x+1)^{5}}{(x-1)^{2}}}dx \quad (222$$

$$\int \frac{1}{\sqrt{(x-1)^{3}(x-2)}}dx \quad (223$$

$$\int \frac{1}{x+\sqrt{x^{2}+x+1}}dx \quad (224$$

$$\int \frac{1}{\sqrt[3]{x\,2}\left(1+\sqrt{x}\right)}dx \quad (225$$

$$\int x^{5}\left(3+2x^{2}\right)^{-\frac{3}{2}}dx \quad (226$$

$$\int \left(nx\right)^{\frac{1-n}{x}}dx \quad (227$$

$$\int \left(a^{\frac{2}{3}}-x^{\frac{2}{3}}\right)^{3}dx \quad (228$$

$$\int \left(\sqrt{x}+1\right)\left(x-\sqrt{x}+1\right)dx \quad (229$$

$$\int \frac{\left(x^{2}+1\right)\left(x^{2}-2\right)}{\sqrt[3]{x^{2}}}dx \quad (230$$

$$\int \frac{\left(\sqrt{a}-\sqrt{x}\right)^{4}}{\sqrt{ax}}dx \quad (231$$

$$\int \frac{\sqrt{2+x^{2}}-\sqrt{2-x^{2}}}{\sqrt{4-x^{4}}}dx \quad (232$$

$$\int \frac{e^{2x}}{\sqrt{e^{x}+1}}\,dx \quad (233$$

$$\int \frac{1}{(x-1)\sqrt{x^{2}-3x+2}}\,dx \quad (234$$

$$\int \frac{1}{\left(1+x^{2}\right)\sqrt{1-x^{2}}}\,dx \quad (235$$

$$\int x^{2}e^{2x-1}\,dx \quad (236$$

$$\int \frac{1-\sin x+\cos x}{1+\sin x-\cos x}\,dx \quad (237$$

$$\int \cos\frac{x}{2}\cos\frac{x}{3}\,dx \quad (238$$

$$\int x\sin^{2}\left(x^{2}\right)dx \quad (239$$

$$\int \sin x\sin 2x\sin 3x\,dx \quad (240$$

$$\int \cos\frac{2x}{3}\cos\frac{4x}{3}\,dx \quad (241$$

$$\int \frac{x^4 - 6x^3 + 12x^2 + 6}{x^3 - 6x^2 + 12x - 8} dx \quad (242$$

$$\int \frac{1}{x^8 + x^6} dx \quad (243$$

$$\int \frac{\sqrt[3]{1 + \sqrt[4]{x}}}{\sqrt{x}} dx \quad (244$$

$$\int \frac{\ln x}{x \sqrt{1 - 4\ln x - \ln^2 x}} dx \quad (245$$

$$\int \frac{1 + \tan x}{1 - \tan x} dx \quad (246$$

$$\int \frac{shx}{\sqrt{ch\,2x}} dx \quad (247$$

$$\int \frac{1}{thx - 1} dx \quad (248$$

$$\int x^2 \cos^3\left(x^3\right) dx \quad (249$$

$$\int \frac{1}{2shx + 3chx} dx \quad (250$$

$$\int \sin 3x \sqrt[3]{\cos x}\, dx \quad (251$$

$$\int \left(1+\sqrt{x}\right)^{4} dx \quad (252$$

$$\int e^{-x} \sin 4x \, dx \quad (253$$

$$\int \frac{e^{2t}}{\left(e^{2t}+3\right)^{4}} dt \quad (254$$

$$\int (x+1) \sqrt[3]{\frac{(x+1)^{2}}{(x-1)^{2}}} dx \quad (255$$

أدرس تقارب كل من التكاملات التالية واحسب قيمة التكاملات المتقاربة منها:

257) $$\int_{0}^{4} \frac{1}{\sqrt[3]{x-1}}\,dx$$

258) $$\int_{0}^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{1-\sin x}}\,dx$$

259) $$\int_{-\infty}^{\infty} \frac{1}{x^{2}+4}\,dx$$

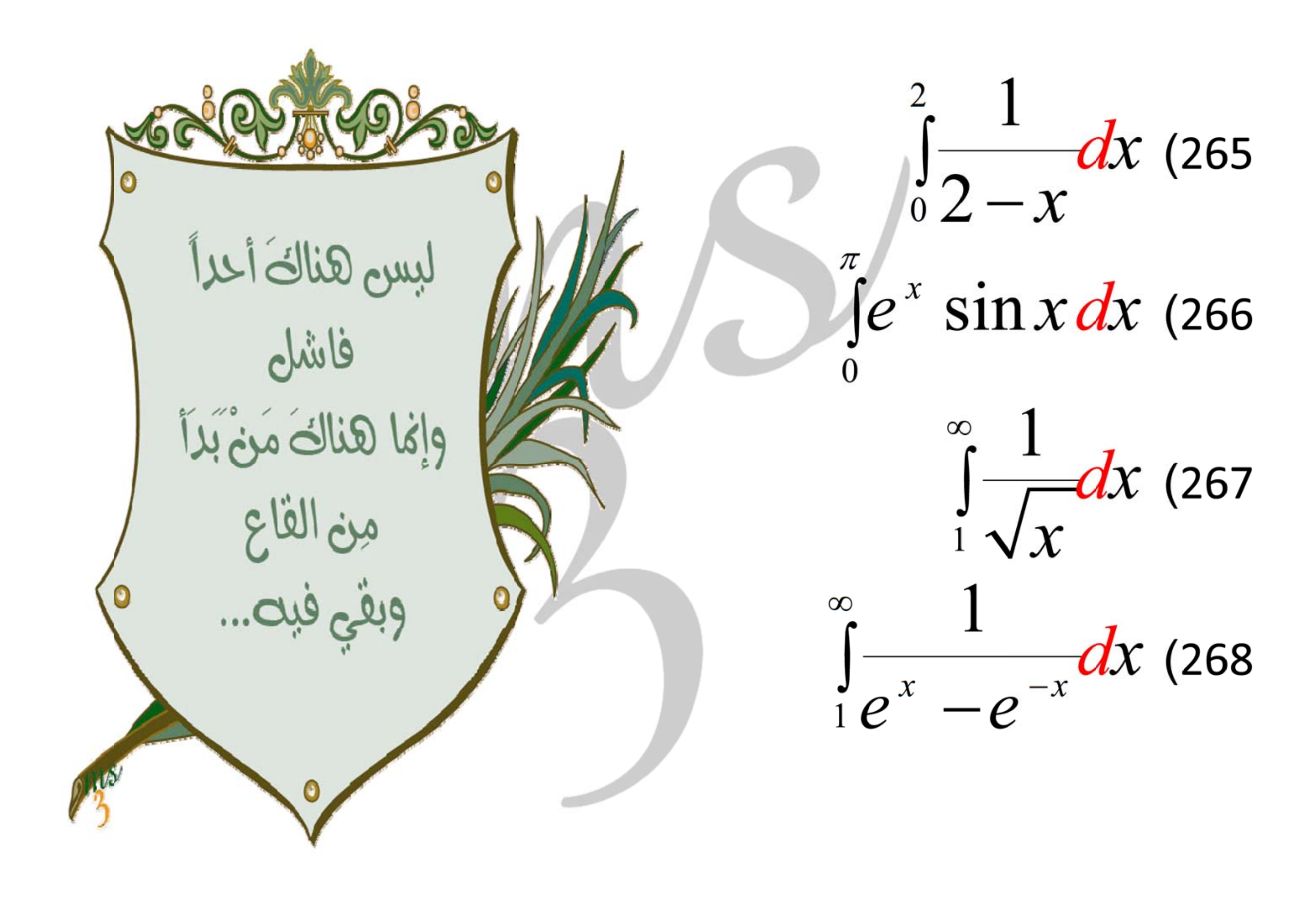
260) $$\int_{-\infty}^{0} e^{2x}\,dx$$

261) $$\int_{1}^{\infty} \frac{1}{x^{2}\left(1+e^{x}\right)}\,dx$$

262) $$\int_{1}^{\infty} \frac{x+1}{\sqrt{x^{3}}}\,dx$$

263) $$\int_{0}^{3} \frac{1}{\sqrt{9-x^{2}}}\,dx$$

264) $$\int_{0}^{4} \frac{1}{(x-1)^{2}}\,dx$$



$$\int_0^2 \frac{1}{2-x}\,dx \quad (265$$

$$\int_0^\pi e^x \sin x\,dx \quad (266$$

$$\int_1^\infty \frac{1}{\sqrt{x}}\,dx \quad (267$$

$$\int_1^\infty \frac{1}{e^x - e^{-x}}\,dx \quad (268$$

أحسب التكاملات المحددة التالية :

269) $\displaystyle\int_{-1}^{2} \frac{|x|}{x^2+1}\,dx$

270) $\displaystyle\int_{1}^{e} \left(\frac{\ln x}{x} - \frac{2}{x} + x + 1 \right) dx$

271) $\displaystyle\int_{0}^{\frac{\pi}{2}} x \cos x \, dx$

272) $\displaystyle\int_{0}^{1} x e^{-x} \, dx$

273) $\displaystyle\int_{\frac{1}{2}}^{1} x^2 \ln 2x \, dx$

274) $\displaystyle\int_{\frac{\sqrt{2}}{2}}^{1} \frac{\sqrt{1-x^2}}{x^2}\,dx$

275) $\displaystyle\int_{\ln 2}^{\ln 3} \frac{1}{ch^2 x}\,dx$

$$\int_{2}^{5} \frac{1}{\sqrt{x^{6}+4}}\,dx \quad (280$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan x\,dx \quad (281$$

$$\int_{0}^{1} \frac{1}{\sqrt{1-x^{4}}}\,dx \quad (282$$

$$\int_{0}^{\pi/2} \frac{1}{2\cos x+3}\,dx \quad (283$$

$$\int_{3}^{29} \frac{\sqrt[3]{(x-2)^{2}}}{\sqrt[3]{(x-2)^{2}}+3}\,dx \quad (276$$

$$\int_{0}^{1} x^{3}e^{2x}\,dx \quad (277$$

$$\int_{0}^{1} \frac{1}{x^{2}+4x+5}\,dx \quad (278$$

$$\int_{1}^{e} \frac{\sin(\ln x)}{x}\,dx \quad (279$$

$$\int_0^{\pi} e^{x} \sin x \, dx$$ (284

$$\int_0^{\pi} \frac{e^{x}\sqrt{e^{x}-1}}{e^{x}+3} dx$$ (285

$$\int_0^{1} dx$$ (286

حساب مساحة منحني لتابع معطى بالإحداثيات الديكارتية:

287) أحسب مساحة السطح المحدد بمنحني التابع

$y = x^2 + 2x + 2$ والمحور ox و $x = -2$ و $x = 1$

288) أحسب مساحة السطح المحصور بين منحني التابع $y = e^x$ والمحور ox والمستقيمين $x = 1$ و $x = 2$

289) أحسب مساحة السطح المحصور بين منحني التابع $y = \sin x$ والمحور ox و $x = 2\pi$ و $x = 0$

290) أحسب مساحة السطح المحددة بالقطع المكافئ $x = 8 + 2y - y^2$ ومحور التراتيب والمستقيمات $y = 3$ و $y = -1$

291) أحسب مساحة السطح المحدد بالمنحنيين $y = \sqrt{x}$ و $y = x$

292) أحسب مساحة السطح المحدد بالمنحنيين $y = -x^2 + 4x + 8$ و $y = x^2 - 2x + 9$.

293) أحسب مساحة الجزء المشترك بين الدائرتين $x^2+y^2=4x$ و $x^2+y^2=4$.

294) أحسب مساحة السطح المحدد بالقطع الزائد $xy=a^2$ ومحور السينات والمستقيمين $x=a$ و $x=2a$.

295) أحسب مساحة السطح المحصور بين المنحني $y=x^3$ و $y=2x$.

296) أحسب مساحة السطح المحصور بين الدائرة $x^2+y^2=4ax$ والقطع الناقص $y^2=2ax$.

297) أحسب مساحة السطح المحدد بالقطع الناقص $y=b\sin t$ و $x=a\cos t$.

298) أحسب مساحة السطح المحدد بالمنحني $x=3+\cos\theta$ و $y=4\sin\theta$

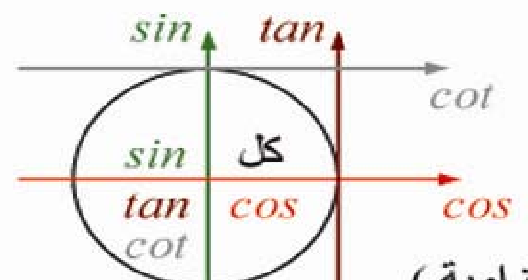
حساب مساحة منحني لتابع معطى بالإحداثيات القطبية:

299) أحسب مساحة السطح داخل القطع الناقص $y=b\sin t$ و $x=a\cos t$.

300) أحسب مساحة الاسترويئيد المعرف بالمعادلتين

$y=a\sin^2 t$ و $x=a\cos^2 t$

تذكـرة ببعض القوانين الـهامـة:



الارجاع الى الربع الأول

2π ، π لا تغير النسبة المثلثية

$\frac{\pi}{2}$ ، $\frac{3\pi}{2}$ تغير النسبة المثلثية (مع الانتباه للإشارة بحسب الربع الذي تقع فيه الزاوية)

* ملاحظة هامة : إذا كان مجموع زاويتين $90°$ فإن : sin إحداهما = cos الأخرى ، tan إحداهما = cot الأخرى

$\cos(-x) = \cos x$	$\sin(-x) = -\sin x$	$\tan(-x) = -\tan(x)$	$\cot(-x) = -\cot x$
$-\cos x = \cos(\pi \mp x)$	$-\sin x = \sin(-x)$	$-\tan x = \tan(-x)$	$-\cot = \cot(-x)$
$\cos x = \sin\left(\frac{\pi}{2} - x\right)$		$\sin x = \cos\left(\frac{\pi}{2} - x\right)$	

النسب المثلثية لضعفي زاوية

$$\sin 2\alpha = 2\sin\alpha \cdot \cos\alpha \qquad \tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

$$\cos 2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha = \cos^2\alpha - \sin^2\alpha = (\cos\alpha + \sin\alpha)(\cos\alpha - \sin\alpha)$$

النسب المثلثية لزاوية بدلالة تجيب ضعفيها

$$\cos^2\alpha = \frac{1 + \cos 2\alpha}{2} \qquad \sin^2\alpha = \frac{1 - \cos 2\alpha}{2} \qquad \tan^2\alpha = \frac{1 - \cos 2\alpha}{1 + \cos 2\alpha}$$

النسب المثلثية لثلاثة أمثال زاوية

$$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha \qquad \sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$$

العلاقات الأساسية بين النسب المثلثية

$\sin^2\theta+\cos^2\theta=1$	$\tan\theta=\dfrac{\sin\theta}{\cos\theta}=\dfrac{1}{\cot\theta}$	$1+\tan^2\theta=\dfrac{1}{\cos^2\theta}$	$1+\cot^2\theta=\dfrac{1}{\sin^2\theta}$

دساتير النسب المثلثية لجمع وفرق زاويتين

$\cos(\alpha\pm\beta)=\cos\alpha\cdot\cos\beta\mp\sin\alpha\cdot\sin\beta$	$\sin(\alpha\pm\beta)=\sin\alpha\cdot\cos\beta\pm\cos\alpha\cdot\sin\beta$

$$\tan(\alpha\pm\beta)=\frac{\tan\alpha\pm\tan\beta}{1\mp\tan\alpha\cdot\tan\beta}$$

دساتير التحويل

$2\cos\alpha\cdot\cos\beta=\cos(\alpha+\beta)+\cos(\alpha-\beta)$	$2\sin\alpha\cdot\sin\beta=\cos(\alpha-\beta)-\cos(\alpha+\beta)$
$2\sin\alpha\cdot\cos\beta=\sin(\alpha+\beta)+\sin(\alpha-\beta)$	$2\cos\alpha\cdot\sin\beta=\sin(\alpha+\beta)-\sin(\alpha-\beta)$

$\cos\alpha+\cos\beta=2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$	$\cos\alpha-\cos\beta=-2\sin\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}$
$\sin\alpha+\sin\beta=2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$	$\sin\alpha-\sin\beta=2\cos\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}$

$\tan\alpha\pm\tan\beta=\dfrac{\sin(\alpha\pm\beta)}{\cos\alpha\cdot\cos\beta}$	$\cot\alpha\pm\cot\beta=\dfrac{\sin(\beta\pm\alpha)}{\sin\alpha\cdot\sin\beta}$

$\sin^2\alpha-\sin^2\beta=\sin(\alpha+\beta)\cdot\sin(\alpha-\beta)$	$\cos^2\alpha+\cos^2\beta=\cos(\alpha+\beta)\cdot\cos(\alpha-\beta)+1$

*** تذكرة بالمتطابقات الشهيرة :**

$(a+b)^2=a^2+2a\cdot b+b^2$, $(a-b)^2=a^2-2a\cdot b+b^2$, $a^2-b^2=(a+b)(a-b)$

$(a+b)^3=a^3+3a^2\cdot b+3a\cdot b^2+b^3$, $(a-b)^3=a^3-3a^2\cdot b+3a\cdot b^2-b^3$

$a^3+b^3=(a+b)(a^2-a\cdot b+b^2)$, $a^3-b^3=(a-b)(a^2+a\cdot b+b^2)$

$f(x)$	$\int f(x)dx$	$f(x)$	$\int f(x)dx$
x^n	$\frac{x^{n+1}}{n+1}$ $(n \neq -1)$	$[g(x)]^n g'(x)$	$\frac{[g(x)]^{n+1}}{n+1}$ $(n \neq -1)$
$\frac{1}{x}$	$\ln \lvert x \rvert$	$\frac{g'(x)}{g(x)}$	$\ln \lvert g(x) \rvert$
e^x	e^x	a^x	$\frac{a^x}{\ln a}$ $(a > 0)$
$\sin x$	$-\cos x$	$\sinh x$	$\cosh x$
$\cos x$	$\sin x$	$\cosh x$	$\sinh x$
$\tan x$	$-\ln \lvert \cos x \rvert$	$\tanh x$	$\ln \cosh x$
$\operatorname{cosec} x$	$\ln \left\lvert \tan \frac{x}{2} \right\rvert$	$\operatorname{cosech} x$	$\ln \left\lvert \tanh \frac{x}{2} \right\rvert$
$\sec x$	$\ln \lvert \sec x + \tan x \rvert$	$\operatorname{sech} x$	$2 \tan^{-1} e^x$
$\sec^2 x$	$\tan x$	$\operatorname{sech}^2 x$	$\tanh x$
$\cot x$	$\ln \lvert \sin x \rvert$	$\coth x$	$\ln \lvert \sinh x \rvert$
$\sin^2 x$	$\frac{x}{2} - \frac{\sin 2x}{4}$	$\sinh^2 x$	$\frac{\sinh 2x}{4} - \frac{x}{2}$
$\cos^2 x$	$\frac{x}{2} + \frac{\sin 2x}{4}$	$\cosh^2 x$	$\frac{\sinh 2x}{4} + \frac{x}{2}$

$$\int f(x)g(x)dx = F(x)g(x) - \int F(x)\frac{dg}{dx}dx$$

$f(x)$	$\int f(x)\,dx$	$f(x)$	$\int f(x)\,dx$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\frac{x}{a}$ $(a>0)$	$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\ln\left\|\frac{a+x}{a-x}\right\|$ $(0<\|x\|<a)$
		$\frac{1}{x^2-a^2}$	$\frac{1}{2a}\ln\left\|\frac{x-a}{x+a}\right\|$ $(\|x\|>a>0)$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\frac{x}{a}$ $(-a<x<a)$	$\frac{1}{\sqrt{a^2+x^2}}$	$\ln\left\|\frac{x+\sqrt{a^2+x^2}}{a}\right\|$ $(a>0)$
		$\frac{1}{\sqrt{x^2-a^2}}$	$\ln\left\|\frac{x+\sqrt{x^2-a^2}}{a}\right\|$ $(x>a>0)$
$\sqrt{a^2-x^2}$	$\frac{a^2}{2}\left[\sin^{-1}\left(\frac{x}{a}\right)+\frac{x\sqrt{a^2-x^2}}{a^2}\right]$	$\sqrt{a^2+x^2}$	$\frac{a^2}{2}\left[\sinh^{-1}\left(\frac{x}{a}\right)+\frac{x\sqrt{a^2+x^2}}{a^2}\right]$
		$\sqrt{x^2-a^2}$	$\frac{a^2}{2}\left[-\cosh^{-1}\left(\frac{x}{a}\right)+\frac{x\sqrt{x^2-a^2}}{a^2}\right]$